

## Some no-go theorems for string duals of non-relativistic Lifshitz-like theories

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JHEP10(2009)015

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## Some no-go theorems for string duals of non-relativistic Lifshitz-like theories

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**ABSTRACT:** We study possibilities of string theory embeddings of the gravity duals for non-relativistic Lifshitz-like theories with anisotropic scale invariance. We search classical solutions in type IIA and eleven-dimensional supergravities which are expected to be dual to (2+1)-dimensional Lifshitz-like theories. Under reasonable ansätze, we prove that such gravity duals in the supergravities are not possible. We also discuss a possible physical reason behind this.

**KEYWORDS:** Flux compactifications, D-branes, AdS-CFT Correspondence

**ARXIV EPRINT:** [0908.0363](https://arxiv.org/abs/0908.0363)

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## 1 Introduction

Recently, two interesting extensions of the AdS/CFT correspondence [1] to non-relativistic systems were proposed in the papers [2] and [3]. The former has the non-relativistic conformal symmetry including the special conformal transformation, while the latter, called quantum Lifshitz-like points, only has the non-relativistic scale invariance.<sup>1</sup>

To understand their microscopic holographic dual theories, we need to find their concrete string theory realizations. Another reason for this is that string embeddings are expected to give a consistency condition for solutions of an effective gravity theory which lives in the lower dimensions via compactifications [24]. The original gravity duals for the non-relativistic systems have been proposed in effective gravity-matter theories living in  $d < 10$  (or  $d < 11$ ) dimension [2, 3], which are not a priori guaranteed to be derived from ten (or eleven) dimensional supergravity. The effective  $d$ -dimensional spacetime dual to a  $(d - 1)$ -dimensional Lifshitz-like theory will be called  $Li_d$  in this paper. Later, it has been found that the gravity duals of the former (i.e. non-relativistic conformal) theories can be realized as solutions in ten or eleven dimensional supergravities as was first done

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<sup>1</sup>For recent discussions of the Lifshitz-like points, refer to [4–22]. See also the seminal paper [23] for the quantum field theoretic properties of the Lifshitz-like points.

in [25]. However, the presence of a null circle in the gravity dual [2] of non-relativistic conformal systems makes the connection to interesting condensed matter systems (such as cold atoms) ambiguous.

The main purpose of this paper is to point out that it is surprisingly difficult to embed the gravity duals of quantum Lifshitz-like points [3] into ten or eleven dimensional supergravities as opposed to our naive expectation. Actually, the only known supergravity solution with the Lifshitz-like property is the one in the paper [14]. It is based on the D3-D7 system first introduced in [26] and can be embedded into type IIB string theory. However, in this solution we can introduce the anisotropy of the scale transformation only through one of the three spatial directions therefore it corresponds not to a non-relativistic quantum Lifshitz point but to a classical Lifshitz point.<sup>2</sup> Also since it has a non-constant dilaton, the anisotropic scale invariance only holds at the leading order of interactions, though thermodynamical quantities such as the entropy still respect the scale invariance.

In this paper, we will present no-go arguments under certain ansätze of fluxes, which we expect will not cause the loss of generality significantly. Such ansätze are necessary to determine clearly if physical solutions are possible or not in our analysis. It may be possible that we can extend our no-go arguments if we develop excellent technical devices in near future. Under the ansätze we will prove that no solutions of the form  $Li_4 \times M_7$ , where  $M_7$  is an arbitrary compact manifold, are allowed in the eleven-dimensional supergravity even if we take the possibility of warp factor into account. We will also present a similar no-go argument in the massive type IIA supergravity. Interestingly enough, we often encounter unphysical supergravity solutions, whose fluxes become imaginary-valued. We will also offer a heuristic physical reason why quantum Lifshitz-like theories are difficult to realize in string theory via a holography argument in the final section, which relates the dual theory to certain non-commutative theories.

Though in this paper we always consider the  $(2 + 1)$ -dimensional Lifshitz-like fixed points to perform concrete calculations, we believe that the main conclusions can be applied to other dimensions. As opposed to the proofs of no-go theorems for de-Sitter spaces [28] and flux backgrounds [29] in supergravity, we need more detailed arguments due to the lack of the Lorentz symmetry.

This paper is organized as follows: In section 2, we will review the gravity dual of quantum Lifshitz-like points following the paper [3]. In section 3, we will prove a no-go theorem for the spacetime  $Li_4 \times M_7$  in M-theory assuming a certain ansatz of fluxes. In section 4, we will present a no-go argument for the spacetime  $Li_4 \times M_6$  in massive IIA supergravity under an even form ansatz of fluxes. We will also show the no-go theorem for deformations of standard flux compactifications when  $M_6$  is an arbitrary nearly Kähler manifold. Moreover, we also consider the possibility of adding an orientifold 6-plane (O6-plane) and show that the result will not be changed. In section 5, we will summarize our conclusion and discuss our results.

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<sup>2</sup>In the presence of fundamental string sources, we can also find a type IIB supergravity solution whose metric in the Einstein frame coincides with  $Li_5$  [27]. However, in this solution the dilaton is not constant and it breaks anisotropic scale invariance again.

## 2 Review of gravity duals of Lifshitz-like fixed points

Let us first review the gravity dual  $Li_4$  of Lifshitz-like fixed points in  $2 + 1$  dimension, which was first presented in [3]. We start with the four-dimensional gravity coupled to a one-form gauge field  $A_1$  and a two-form field  $B_2$ . The action is

$$S = \int d^4x \sqrt{-g} (R - 2\Lambda) - \frac{1}{2} \int (F_2 \wedge *F_2 + H_3 \wedge *H_3) - c \int B_2 \wedge F_2. \quad (2.1)$$

The gravity dual  $Li_4$  of a Lifshitz-like point is defined by the following metric

$$ds^2 = L^2 \left( -r^{2z} dt^2 + r^2(dx^2 + dy^2) + \frac{dr^2}{r^2} \right) \quad (2.2)$$

$$\equiv -(\theta^t)^2 + (\theta^x)^2 + (\theta^y)^2 + (\theta^r)^2, \quad (2.3)$$

where we defined the orthonormal basis of one-forms

$$\theta^t = Lr^z dt, \quad \theta^x = Lr dx, \quad \theta^y = Lr dy, \quad \theta^r = L \frac{dr}{r}. \quad (2.4)$$

The Ricci tensor  $R_{\mu\nu}$  and the Ricci scalar  $R_4$  is given by

$$R_{tt} = z(z+2)r^{2z}, \quad R_{xx} = R_{yy} = -(2+z)r^2, \quad R_{rr} = -\frac{2+z^2}{r^2}. \quad (2.5)$$

$$R_4 = -\frac{2}{L^2}(3+2z+z^2). \quad (2.6)$$

In order to break the  $(2+1)$ -dimensional Lorentz symmetry, we introduce the background fluxes

$$F_2 = \alpha \theta^t \wedge \theta^r, \quad H_3 = \beta \theta^x \wedge \theta^y \wedge \theta^r. \quad (2.7)$$

Here the fact that the fluxes are written by the orthonormal one-forms (2.4) guarantees the non-relativistic scale invariance and is consistent with the Einstein equations discussed below.

In this system the equations of motion for fluxes are given by

$$d * F_2 = -c H_3, \quad d * H_3 = c F_2. \quad (2.8)$$

This leads to the constraints

$$\frac{\alpha}{\beta} = \sqrt{\frac{z}{2}}, \quad c^2 = \frac{2z}{L^2}. \quad (2.9)$$

The Einstein equation becomes

$$R_{\mu\nu} + g_{\mu\nu} \left( -\frac{R}{2} + \frac{1}{24} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} + \frac{1}{8} F_{\rho\sigma} F^{\rho\sigma} + \Lambda \right) = \frac{1}{2} F_{\mu\alpha} F_{\nu}^{\alpha} + \frac{1}{4} H_{\mu\alpha\beta} H_{\nu}^{\alpha\beta}. \quad (2.10)$$

By substituting the Ricci tensor (2.5) and the flux ansatz (2.7) into (2.10), we obtain in the end

$$\alpha = \frac{\sqrt{2z(z-1)}}{L}, \quad \beta = \frac{2}{L} \sqrt{z-1}, \quad \Lambda = -\frac{z^2+z+4}{2L^2}. \quad (2.11)$$

Notice that in this solution the coefficient  $c$  of Chern-Simons term is uniquely determined by the cosmological constant  $\Lambda$ . In the later arguments of its string theory embedding, what turns out to be crucial is whether this constraint is consistent with string theory compactifications or not.

It is also useful to take a duality transformation. We add a term  $\int d\phi \wedge H_3$  assuming that  $H_3$  is now a general three-form. The equation of motion for  $\phi$  leads to the Bianchi identity  $dH_3 = 0$ . If we integrate out  $H_3$ , we find the action looks like

$$S = \int d^4x \sqrt{-g} (R - 2\Lambda) - \frac{1}{2} \int (F_2 \wedge *F_2 + (d\phi - cA_1) \wedge *(d\phi - cA_1)). \quad (2.12)$$

By absorbing  $\phi$  via the gauge transformation, this theory is equivalent to a massive vector field theory coupled to gravity, as already noted in [5].

### 3 No-go theorem for $Li_4 \times M_7$ in M-theory

#### 3.1 Direct product metric $Li_4 \times M_7$

The action of the eleven-dimensional supergravity is given by<sup>3</sup>

$$S = \frac{1}{2\kappa_{11}^2} \left[ \int d^{11}x \sqrt{-g} \left( R - \frac{1}{2 \cdot 4!} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} \right) - \frac{1}{6} \int C_3 \wedge F_4 \wedge F_4 \right]. \quad (3.1)$$

The Bianchi identity and equations of motion of the four-form flux read

$$dF_4 = 0, \quad d * F_4 + \frac{1}{2} F_4 \wedge F_4 = 0, \quad (3.2)$$

where  $F_4 = dC_3$ .

We assume that the spacetime metric takes the form of the direct product  $Li_4 \times M_7$  without any warp factor, where  $M_7$  is taken to be an arbitrary seven-dimensional compact manifold. The coordinates of  $M_7$  are denoted by  $x^i = (x^1, x^2, \dots, x^7)$ . We denote the volume-form of  $M_7$  by  $V_7$ . Now we take the the four-form flux with the symmetry of  $Li_4$  to be in the following form

$$F_4 = f \theta^t \theta^x \theta^y \theta^r + \theta^x \theta^y \theta^r \wedge \alpha + \theta^t \theta^r \wedge \beta + \theta^r \wedge \Omega + \eta. \quad (3.3)$$

This form of flux is the most general form in which we can tell precisely if the corresponding supergravity solution exists or not, using the arguments presented below. Its Hodge dual is given by<sup>4</sup>

$$*F_4 = -fV_7 + \theta^t \wedge *\alpha - \theta^x \theta^y \wedge *\beta + \theta^t \theta^x \theta^y \wedge *\Omega + \theta^t \theta^x \theta^y \theta^r \wedge *\eta. \quad (3.4)$$

The Einstein equation requires that the  $r$ -dependence in  $F_4$  only appears through  $\theta^\mu$  as in the previous section. Thus,  $f$ ,  $\alpha$ ,  $\beta$ ,  $\Omega$  and  $\eta$  are 0,1,2,3 and 4-form in  $M_7$  and they do not depend on  $r$ .

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<sup>3</sup> In this paper, we will adopt the normalization of supergravity in [30].

<sup>4</sup> In our convention, the epsilon tensor is chosen to be  $\epsilon_{txyr1234567} = -\epsilon^{txyr1234567} = 1$ , which leads to the relation  $\frac{1}{p!} F^{\mu_1, \dots, \mu_p} F_{\mu_1, \dots, \mu_p} V_7 = F \wedge *F$ .

The Bianchi identity requires

$$d\alpha = d\beta = d\Omega = df = d\eta = 0. \quad (3.5)$$

The flux equations of motion lead to

$$d * \alpha = d * \beta = d * \Omega = 0, \quad (3.6)$$

$$\beta \wedge \eta = \frac{z}{L} * \alpha, \quad (3.7)$$

$$\alpha \wedge \eta = \frac{2}{L} * \beta, \quad (3.8)$$

$$d * \eta - \frac{z+2}{L} * \Omega + \eta f = 0, \quad (3.9)$$

$$\Omega \wedge \eta = 0. \quad (3.10)$$

In particular, from (3.7) and (3.8), we find

$$z\alpha^2 = \beta^2, \quad (3.11)$$

where we have defined  $\alpha^2 = \alpha_i \alpha^i$  and  $\beta^2 = \beta_{ijk} \beta^{ijk}$  with  $i = 1, 2, \dots, 7$ .

The Einstein equation can be rewritten into the following form

$$R_{\mu\nu} = -\frac{1}{6 \cdot 4!} F^2 g_{\mu\nu} + \frac{1}{12} F_{\mu\rho\sigma\tau} F_{\nu}{}^{\rho\sigma\tau}. \quad (3.12)$$

Therefore, the Einstein equation (3.12) for the  $(tt)$ ,  $(xx)$  and  $(rr)$  components reads

$$\begin{aligned} -\frac{z(z+2)}{L^2} &= -\frac{1}{6} \left( -f^2 + \alpha^2 - \frac{1}{2}\beta^2 + \frac{1}{3!}\Omega^2 + \frac{1}{4!}\eta^2 \right) + \frac{1}{2} \left( -f^2 - \frac{1}{2}\beta^2 \right), \\ -\frac{2+z}{L^2} &= -\frac{1}{6} \left( -f^2 + \alpha^2 - \frac{1}{2}\beta^2 + \frac{1}{3!}\Omega^2 + \frac{1}{4!}\eta^2 \right) + \frac{1}{2}(-f^2 + \alpha^2), \\ -\frac{2+z^2}{L^2} &= -\frac{1}{6} \left( -f^2 + \alpha^2 - \frac{1}{2}\beta^2 + \frac{1}{3!}\Omega^2 + \frac{1}{4!}\eta^2 \right) + \frac{1}{2} \left( -f^2 + \alpha^2 - \frac{1}{2}\beta^2 + \frac{1}{6}\Omega^2 \right). \end{aligned}$$

By taking differences among them, we can easily find

$$\frac{2(z-1)}{L^2} = \frac{1}{2}\alpha^2 + \frac{1}{12}\Omega^2, \quad \frac{z(z-1)}{L^2} = \frac{\beta^2}{4} - \frac{1}{12}\Omega^2. \quad (3.13)$$

By combining these two equations with the relation (3.11), we find if  $\Omega^2 \neq 0$ , then  $z = -2$ . In this case, we only have unphysical solutions that has  $\frac{\beta^2}{\alpha^2} = -2 < 0$ . Therefore, we need to require  $\Omega^2 = 0$  (or equivalently  $\Omega_{ijk} = 0$ ) below.

Now let us look at the off-diagonal components of the Einstein equation. In particular, if we consider  $(ti)$  component it behaves like  $R_{ti} \propto f\alpha_i$  and this contradicts our metric ansatz unless it is vanishing. If we assume  $f \neq 0$ , then we need to set  $\alpha_i = 0$  and  $z = 1$ . Thus we recover the full Lorentz symmetry, which we do not want.

Thus, we need to set  $f = 0$ . In this case, all equations of motion can be reduced to

$$\frac{2z^2 + 12z + 4}{L^2} = \frac{\eta^2}{4!}, \quad (3.14)$$

$$R_{ij} = -\frac{3z}{L^2} g_{ij} + \frac{1}{2}(\alpha_i \alpha_j - \beta_{ik} \beta_j^k + \frac{1}{6} \eta_{iklm} \eta_j^{klm}), \quad (3.15)$$

$$d\alpha = d\beta = d\eta = 0, \quad f = \Omega = 0, \quad d*\alpha = d*\beta = d*\eta = 0, \quad (3.16)$$

$$\beta \wedge \eta = \frac{z}{L} * \alpha, \quad \alpha \wedge \eta = \frac{2}{L} * \beta, \quad (3.17)$$

$$\alpha^2 = \frac{\beta^2}{z} = \frac{4(z-1)}{L^2}. \quad (3.18)$$

We can also find that the value of the Ricci scalar of  $M_7$  as  $R_M = \frac{(2z+3)(z+2)}{L^2} > 0$ .

By using (3.17) we can find the relation

$$\alpha \wedge *\alpha = \frac{L^2}{2z} * (\alpha \wedge \eta) \wedge (\alpha \wedge \eta). \quad (3.19)$$

This equation (3.19) can be rewritten in the component expression as

$$\alpha^2 = \frac{25L^2}{2z \cdot 5!} \alpha_{[\mu_1} \eta_{\mu_2 \mu_3 \mu_4 \mu_5]} \alpha^{[\mu_1} \eta^{\mu_2 \mu_3 \mu_4 \mu_5]} \quad (3.20)$$

$$= \frac{L^2}{2z \cdot 4!} \left( \alpha^2 \eta^2 - 4\alpha^i \alpha_j \eta_{iklm} \eta^{jklm} \right). \quad (3.21)$$

By plugging into (3.18) we obtain

$$\frac{1}{4!} \alpha^i \alpha_j \eta_{iklm} \eta^{jklm} = \frac{2(z-1)(z^2+5z+2)}{L^4} > 0. \quad (3.22)$$

On the other hand, we can show  $\alpha^i \beta_{ij} = 0$  as we can easily see  $\alpha \wedge *\beta = 0$  from (3.17). By combining these results, we can finally evaluate

$$R_{ij} \alpha^i \alpha^j = \frac{4z(z-1)(z+4)}{L^4} > 0, \quad (3.23)$$

which actually proves that there is no such harmonic one-form on  $M_7$ . This statement follows from the Weitzenbock formula (see e.g. [31])

$$\alpha^i \Delta \alpha_i = -\alpha^i \nabla_j \nabla^j \alpha_i + R_{ij} \alpha^i \alpha^j, \quad (3.24)$$

where the Laplacian is defined by  $\Delta = d\delta + \delta d$  as usual in the harmonic analysis;  $\delta$  is the codifferential and  $\nabla$  is the covariant derivative. Since a harmonic one-form  $\alpha$  satisfies  $d\alpha = \delta\alpha = 0$ , the left-hand side of (3.24) vanishes. On the other hand, if we integrate the right-hand side on  $M_7$ , it should be positive by the partial integration and the formula (3.23). Therefore, such a harmonic one-form  $\alpha$  cannot exist and this completes our no-go argument for  $Li_4 \times M_7$ .

### 3.2 Warp factor in $Li_4 \times M_7$

Actually, we can still have the freedom of taking the warp factor of the metric into account as follows

$$ds^2 = e^{2A(\xi)} ds_{Li_4}^2 + e^{\frac{4}{5}A(\xi)} ds_{M_7}^2, \quad (3.25)$$

where  $\xi$  denotes the seven coordinates of  $M_7$ . We fixed the power of the warp factor by the reparameterizations such that the one-form  $\alpha$  in  $M_7$  again becomes harmonic  $d\alpha = d*\alpha = 0$



due to the flux equation of motion. The equations of motion for fluxes are only modified by powers of  $e^A$ . For example, we can show

$$\alpha^2 = \frac{\beta^2}{z} e^{\frac{6}{5}A} = 4(z-1)e^{\frac{24}{5}A}. \quad (3.26)$$

We also have to require  $f = \Omega_{ijk} = 0$  again. Moreover, by using these relations, we can eventually evaluate the right-hand side of (3.24) as follows:

$$\begin{aligned} & R_{ij}\alpha^i\alpha^j - \alpha^i\nabla_j\nabla^j\alpha_i \\ &= 4z(z-1)(z+4)e^{\frac{18}{5}A} + \frac{48(z-1)}{5}(\nabla_i\nabla^iA + 6(\nabla_iA)(\nabla^iA))e^{\frac{24}{5}A} + 6\alpha^i\alpha^j\nabla_i\nabla_jA \\ &\quad + (\nabla_i\alpha^j)(\nabla^i\alpha_j) - \nabla_j(\alpha^i\nabla^j\alpha_i) \\ &= 4z(z-1)(z+4)e^{\frac{18}{5}A} - \frac{1152}{25}(z-1)(\nabla_iA)(\nabla^iA)e^{\frac{24}{5}A} + 6\nabla^i(\alpha_i\alpha_j\nabla^jA) + (\nabla_i\alpha^j)(\nabla^i\alpha_j). \end{aligned} \quad (3.27)$$

Now let us recall that  $M_7$  is a compact manifold. Therefore, the function  $A(\xi)$  on  $M_7$  should take its maximum and minimum value somewhere. Suppose it takes the minimum value at  $\xi = \xi_{\min}$ . It is obvious that

$$\partial_i A(\xi_{\min}) = 0, \quad \alpha^i\alpha^j\partial_i\partial_j A(\xi_{\min}) \geq 0. \quad (3.28)$$

On the other hand, since  $\alpha$  is a harmonic one-form, the left-hand side of (3.27) is vanishing as explained before. By evaluating this at  $\xi = \xi_{\min}$ , we obtain

$$0 = 4z(z-1)(z+4)e^{\frac{18}{5}A} + 6\alpha^i\alpha^j\partial_i\partial_j A + (\nabla_i\alpha^j)(\nabla^i\alpha_j). \quad (3.29)$$

This is clearly inconsistent as its right-hand side is positive. Thus, there are no such harmonic one-form  $\alpha$  and this finishes the no-go theorem for the warped case.

## 4 Massive IIA supergravity on $Li_4 \times M_6$

### 4.1 Massive IIA theory

The massive IIA supergravity [32] is obtained by adding a new auxiliary scalar field  $M$  and ten-form RR-flux  $F_{10} = dC_9$  and by shifting the flux  $F_2$  and  $F_4$  such that

$$\begin{aligned} \tilde{F}_2 &= dC_1 - MB_2, \\ \tilde{F}_4 &= dC_3 - A_1 \wedge H_3 + \frac{1}{2}MB_2 \wedge B_2. \end{aligned} \quad (4.1)$$

The action is defined by

$$S_{\text{massive IIA}} = \frac{1}{2\kappa^2} \int \mathcal{L}, \quad (4.2)$$

where

$$\begin{aligned} \mathcal{L} &= \sqrt{-g}e^{-2\phi}(R + 4\partial_\mu\phi\partial^\mu\phi) - \frac{e^{-2\phi}}{2}H_3 \wedge *H_3 - \frac{1}{2}\tilde{F}_2 \wedge *\tilde{F}_2 - \frac{1}{2}\tilde{F}_4 \wedge *\tilde{F}_4 - \frac{1}{2}M \wedge *M \\ &\quad + M \wedge F_{10} - \frac{1}{2}B_2 \wedge dC_3 \wedge dC_3 - \frac{M}{6}dC_3 \wedge B_2 \wedge B_2 \wedge B_2 - \frac{M^2}{40}B_2 \wedge B_2 \wedge B_2 \wedge B_2 \wedge B_2. \end{aligned}$$

The Bianchi identities and equations of motion of the fluxes are summarized as follows:

$$\begin{aligned}
dH_3 &= 0, & dM &= 0, \\
d\tilde{F}_2 &= -MH_3, & d\tilde{F}_4 &= -\tilde{F}_2 \wedge H_3, \\
d * \tilde{F}_2 &= H_3 \wedge * \tilde{F}_4, \\
d * \tilde{F}_4 &= -\tilde{F}_4 \wedge H_3, \\
d(e^{-2\phi} * H_3) &= -M \wedge * \tilde{F}_2 + \frac{1}{2} \tilde{F}_4 \wedge \tilde{F}_4 - \tilde{F}_2 \wedge * \tilde{F}_4.
\end{aligned} \tag{4.3}$$

The equation of motion for  $C_9$  requires that  $M$  is a constant:  $M = m$ . On the other hand, the equation of motion of  $M$  just determines  $F_{10}$  in terms of other fields.

The dilaton equation of motion is

$$R + 4\nabla_\mu \nabla^\mu \phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - 4\nabla_\mu \phi \nabla^\mu \phi = 0. \tag{4.4}$$

The Einstein equation becomes

$$R_{\mu\nu} + \frac{1}{4} g_{\mu\nu} A = -2\nabla_\mu \nabla_\nu \phi + \frac{1}{4} H_{\mu\alpha\beta} H_\nu^{\alpha\beta} + \frac{1}{2} e^{2\phi} \tilde{F}_{\mu\alpha} \tilde{F}_\nu^\alpha + \frac{1}{12} e^{2\phi} \tilde{F}_{\mu\alpha\beta\gamma} \tilde{F}_\nu^{\alpha\beta\gamma}, \tag{4.5}$$

where

$$A \equiv \frac{e^{2\phi}}{4!} \tilde{F}_{\mu\nu\rho\sigma} \tilde{F}^{\mu\nu\rho\sigma} + \frac{e^{2\phi}}{2!} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + e^{2\phi} M^2. \tag{4.6}$$

## 4.2 No-go argument in massive IIA on $Li_4 \times M_6$

Now we would like to see if the spacetime  $Li_4 \times M_6$  can be a classical solution of the ten-dimensional massive IIA supergravity. The form of Ricci tensor and the scale invariance require that the dependence of fluxes on  $r$  all comes from the normalized one-forms  $\theta^{t,x,y,z}$  again.

In this subsection, we take into account only even forms in  $M_6$  in the flux ansatz to make the problem easier. In this case, the general flux ansatz with the required Lorentz symmetry breaking for the Lifshitz spacetime can be written as follows<sup>5</sup>

$$\begin{aligned}
M &= m, \\
\tilde{F}_2 &= \alpha \theta^t \theta^r + \eta \theta^x \theta^y + J_1, \\
H_3 &= \beta \theta^x \theta^y \theta^r, \\
\tilde{F}_4 &= f \theta^t \theta^x \theta^y \theta^r + \theta^x \theta^y J_2 + \theta^t \theta^r J_3 + V_4,
\end{aligned} \tag{4.7}$$

where  $\alpha, \beta$  and  $\eta$  are constants.  $J_1, J_2$  and  $J_3$  are some two-forms on  $M_6$ , and  $V_4$  is a certain four-form on  $M_6$ .

Bianchi identities lead to

$$dJ_1 = dJ_2 = dJ_3 = 0, \quad J_2 = -\frac{\beta L}{2} J_1, \quad m = -\frac{2\eta}{\beta L}, \quad dV_4 = 0. \tag{4.8}$$

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<sup>5</sup>We can show from equations of motion that the term like  $H_3 \sim \theta^r \wedge J$  is not allowed.

The flux equations of motion for  $\tilde{F}_2$  and  $\tilde{F}_4$  lead to

$$d * J_1 = d * J_3 = 0, \quad d * V_4 = 0, \quad f\beta L = 2\alpha, \quad \beta L V_4 = 2 * J_3. \quad (4.9)$$

Below we write  $J_1 \equiv J$  and  $J_3 \equiv K$ . Notice that  $J_2 = -\frac{\beta L}{2}J$  and  $V_4 = \frac{2}{\beta L} * K$ .

Notice that the presence of a non-trivial warp factor  $\Omega(\xi)$  (which depends on the coordinates  $\xi$  of  $M_6$ ) in the metric  $ds^2 = \Omega^2(ds_{L_{i_4}}^2 + ds_{M_6}^2)$  contradicts with one of the flux equations of motion that reads  $f\beta L = 2\alpha\Omega^4(\xi)$ . Therefore, in this setup we can set  $\Omega = 1$ , namely the metric is in the form of the direct product.

If we plug in the explicit values of the fluxes, the  $(tt)$ ,  $(xx)$  and  $(rr)$  components of Einstein equation are expressed as follows:

$$-\frac{z(z+2)}{L^2} = -\frac{A}{4} - \frac{g_s^2}{2}\alpha^2 - \frac{2g_s^2\alpha^2}{\beta^2 L^2} - \frac{g_s^2 K^2}{4}, \quad (4.10)$$

$$-\frac{z+2}{L^2} = -\frac{A}{4} - \frac{2g_s^2\alpha^2}{\beta^2 L^2} + \frac{g_s^2\beta^2 L^2 J^2}{16} + \frac{g_s^2 m^2 L^2 \beta^2}{8} + \frac{\beta^2}{2}, \quad (4.11)$$

$$-\frac{z^2+2}{L^2} = -\frac{A}{4} - \frac{g_s^2}{2}\alpha^2 - \frac{2g_s^2\alpha^2}{\beta^2 L^2} - \frac{g_s^2 K^2}{4} + \frac{\beta^2}{2}, \quad (4.12)$$

with

$$A = g_s^2 \left[ -\frac{4\alpha^2}{\beta^2 L^2} + \frac{J^2 \beta^2 L^2}{8} - \frac{K^2}{2} + \frac{2K^2}{\beta^2 L^2} - \alpha^2 + \frac{m^2 L^2 \beta^2}{4} + \frac{J^2}{2} + m^2 \right]. \quad (4.13)$$

By taking the differences, we immediately obtain

$$\beta^2 = \frac{4(z-1)}{L^2}, \quad (4.14)$$

$$\frac{z(z-1)}{L^2} = \frac{g_s^2 m^2 L^2 \beta^2}{8} + \frac{g_s^2 \alpha^2}{2} + \frac{g_s^2 \beta^2 L^2 J^2}{16} + \frac{g_s^2 K^2}{4}. \quad (4.15)$$

On the other hand, after combined with (4.14) and (4.15), the equation (4.10) actually leads to  $z = -4$ . This shows that we can only have unphysical solutions as the equation (4.14) implies that the  $H$ -flux becomes imaginary. In this way, we have found that under this ansatz, there is no physical solution corresponding to the Lifshitz-like fixed point in the IIA massive supergravity.

Moreover, we can find explicit unphysical solutions. Suppose that  $M_6$  is Kähler-Einstein and that the fluxes are given by the parameters

$$m = \eta = 0, \quad J_1 = -\frac{2k}{R^2}\omega, \quad J_2 = \frac{kL\beta}{R^2}\omega, \quad J_3 = V_4 = 0, \quad (4.16)$$

where  $R$  is the radius of  $M_6$  such that  $R_{ij} = \frac{8}{R^2}g_{ij}$ . The Kähler form  $\omega$  is a harmonic two-form, which satisfies  $g^{kl}\omega_{ik}\omega_{jl} = g_{ij}$ . Then we can show that all equations of motion are satisfied if we set

$$\alpha^2 = \frac{35}{2g_s^2 L^2}, \quad \beta^2 = -\frac{20}{L^2}, \quad k^2 = -\frac{6L^2}{g_s^2}, \quad R^2 = 4L^2, \quad z = -4. \quad (4.17)$$

This requires that  $\beta$  and  $k$  to be imaginary. This argument includes Lorentz symmetry breaking deformations of the  $AdS_4 \times CP^3$  solution dual to the  $\mathcal{N} = 6$  Chern-Simons gauge theory [33]. Notice that this is a rather basic setup as in this model all moduli are stabilized.

Finally, it is also possible to check that the situation will not be improved even if we insert spacetime filling D-branes i.e. non-BPS D9-branes<sup>6</sup> whose action is proportional to  $\int d^9 x e^{-\phi} \sqrt{g}$  setting the tachyon field vanishing.

### 4.3 No-go argument for nearly Kähler compactifications

To generalize our previous no-go argument of Lifshitz-like solutions, we need to take into account odd forms in the compact six-dimensional manifold  $M_6$ . Since a general argument in this case turns out to be quite complicated, we would like to assume the case where  $M_6$  is a nearly Kähler manifold, including Calabi-Yau manifolds. This is motivated by the expectation that the Lifshitz-like solutions will appear by deforming the  $AdS_4 \times M_6$  solution to the massive IIA supergravity. The backgrounds  $AdS_4 \times M_6$  have been intensively studied in the context of flux compactifications (see e.g. [35–39]). One major way to maintain the  $N = 1$  supersymmetry after the compactification is to assume that  $M_6$  is a nearly Kähler manifold [35, 38, 39], which is known to be Einstein. Thus, we believe that this restriction to nearly Kähler manifolds will not lose the generality seriously.

When  $M_6$  is nearly Kähler, the two-form  $J$  that defines the almost complex structure and the holomorphic three-form  $\Omega$  satisfies

$$\Omega \wedge J = 0, \tag{4.18}$$

$$V_6 = \text{Re}\Omega \wedge \text{Im}\Omega = \frac{1}{3!} J \wedge J \wedge J, \tag{4.19}$$

$$d\text{Re}\Omega = 0, \tag{4.20}$$

$$dJ = b\text{Re}\Omega, \tag{4.21}$$

$$d\text{Im}\Omega = -\frac{b}{6} J \wedge J. \tag{4.22}$$

The constant  $b$  measures the extent to which the metric deviates from the Calabi-Yau condition and is relate to the Ricci curvature via

$$R_{ij} = \frac{5}{36} b^2 g_{ij}. \tag{4.23}$$

Since it is straightforward to show that there is no physical solution when  $b = 0$  (i.e. when  $M_6$  is Calabi-Yau), we will restrict to the values  $b \neq 0$  below.

By employing  $J$  and  $\Omega$ , we can write down the flux ansatz<sup>7</sup> with the expected Lorentz symmetry breaking as follows:<sup>8</sup>

$$\begin{aligned} M &= m, \\ \tilde{F}_2 &= aJ + \alpha\theta^t\theta^r + \eta\theta^x\theta^y, \end{aligned}$$

<sup>6</sup>For the definition of non-BPS D-branes refer to e.g. the review article [34].

<sup>7</sup>In this convention the known  $AdS_4 \times M_6$  solution [35, 38, 39] corresponds to  $z = 1$ ,  $\eta = g = \alpha = \beta = h = q = 0$ ,  $a = -\frac{m}{\sqrt{15}}$ ,  $s = \frac{3}{5}m$ ,  $f = -\frac{3\sqrt{3}}{\sqrt{5}}m$ ,  $b = \frac{\sqrt{15}}{L}$ ,  $k = \frac{1}{L}$ .

<sup>8</sup>Here the term  $\theta^r \wedge J$  is not allowed in  $H_3$  as we assume  $dJ \neq 0$ ; the term like  $\theta^r \wedge \text{Re}\Omega$  is not included in  $\tilde{F}_4$  as it is not consistent with the equation of motion  $d * F_2 = H_3 \wedge * \tilde{F}_4$ .

$$\begin{aligned}
H_3 &= \beta\theta^x\theta^y\theta^r + k\text{Re}\Omega, \\
\tilde{F}_4 &= f\theta^t\theta^x\theta^y\theta^r + g\theta^x\theta^y \wedge J + h\theta^t\theta^r \wedge J + q\theta^t \wedge \text{Re}\Omega + \frac{s}{2}J^2.
\end{aligned}
\tag{4.24}$$

The Bianchi identities lead to

$$\eta = -\frac{m\beta L}{2}, \quad ab + km = 0, \tag{4.25}$$

$$g = -\frac{\beta a L}{2}, \quad gb + \eta k = 0, \quad hb - \frac{zq}{L} + \alpha k = 0. \tag{4.26}$$

The equations of motion lead to

$$\frac{2\alpha}{L} = f\beta + kq, \tag{4.27}$$

$$kf - \beta q + sb = 0, \quad -\frac{h}{L} + \frac{bq}{6} + \frac{s\beta}{2} = 0, \tag{4.28}$$

$$\frac{z\beta}{Lg_s^2} + m\eta - 3hs + 3ga - \alpha f = 0, \tag{4.29}$$

$$\frac{ma}{2} - \frac{fs}{2} - gh + sa - \frac{\alpha h}{2} + \frac{g\eta}{2} - \frac{bk}{6g_s^2} = 0, \tag{4.30}$$

$$m\alpha + 3gs + 3ha + f\eta = 0. \tag{4.31}$$

Next we would like to write down the Einstein equations.<sup>9</sup> The useful identities are

$$J_{ik}J_{jl}g^{kl} = g_{ij}, \quad \text{Re}\Omega_{ikl}\text{Re}\Omega_j{}^{kl} = g_{ij}. \tag{4.32}$$

The Einstein equation can be eventually expressed as follows after taking suitable linear combinations

$$-\frac{z(z+2)}{L^2} = -\frac{g_s^2}{4}(\alpha^2 + f^2 + 3g^2 + 3h^2 + q^2 + 3s^2 + 3a^2) - \frac{g_s^2\eta^2}{4} - \frac{g_s^2m^2}{4}, \tag{4.33}$$

$$\frac{z(z-1)}{L^2} = \frac{g_s^2}{2}(\alpha^2 + 3h^2 + 3g^2) + \frac{g_s^2\eta^2}{2}, \tag{4.34}$$

$$\frac{2(z-1)}{L^2} = \frac{g_s^2}{2}q^2 + \frac{\beta^2}{2}. \tag{4.35}$$

If  $q = 0$ , then the situation becomes simpler and become the same as the previous subsection and we immediately find  $z = -4$ . In this case, there are no physical solutions. Thus, we need to set  $q \neq 0$  below.

Now the remaining equations of motion which we have to impose are the dilaton equation and the Einstein equation in  $M_6$  direction. They are equivalent to

$$-\frac{2(z^2 + 2z + 3)}{L^2} + R_6 = \frac{\beta^2 + k^2}{2}, \tag{4.36}$$

$$0 = \frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{3g_s^2}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{g_s^2}{48}\tilde{F}_{\mu\nu\rho\lambda}\tilde{F}^{\mu\nu\rho\lambda} - \frac{5}{2}g_s^2m^2$$

---

<sup>9</sup>Notice that we do not have to worry the unwanted cross terms in the Einstein eq. such as  $R_{ti}$  and  $R_{ri}$  as  $J_{ij}\Omega_k{}^{ij} = 0$ .

$$\begin{aligned}
&= \beta^2 + k^2 - \frac{3g_s^2}{2}(3a^2 - \alpha^2 + \eta^2) \\
&\quad - \frac{g_s^2}{2}(-f^2 + 3g^2 - 3h^2 - q^2 + 3s^2) - \frac{5}{2}g_s^2 m^2.
\end{aligned}$$

By combining these with previous equations we can actually show

$$3L^2k^2 + 3L^2\beta^2 = -6 + z - 2z^2. \tag{4.37}$$

Since the right-hand side is clearly negative when  $z > 1$ , we cannot construct any physical solutions.

#### 4.4 Adding orientifold plane

Adding orientifold planes in general enlarges the set of possible solutions in the flux compactification. For example, adding an O6 plane to the IIA string flux compactification allows supersymmetric  $AdS_4 \times M_6$  solutions with all moduli stabilized and with  $M_6$  being half-flat [35–39]. Supersymmetry requires an O6 to wrap the entire  $AdS_4$  and a supersymmetric three-cycle inside  $M_6$ . Moreover, smearing an O6 inside  $M_6$  allows solutions with  $M_6$  being Calabi-Yau three-fold. For our present purpose, since we do not require Lorentz symmetry, more generic O6 configurations should in principle be allowed (for example, with an O6 wrapping only  $(txy)$ -direction and a four-cycle inside  $M_6$ ). However, for simplicity and for parallel comparison with the  $AdS_4$  case, we will only consider the configuration of an O6 wrapping the entire  $Li_4$  and a three-cycle inside  $M_6$ .

In this subsection, we will show that adding an O6 plane wrapping the entire  $Li_4$  and a three-cycle inside  $M_6$  will not help evade the no-go theorem in the case of  $Li_4 \times M_6$  with  $M_6$  being nearly Kähler. To warm up, we first show that even in the presence of a smeared O6 plane, there exist no Lifshitz-like solutions when  $M_6$  is a Calabi-Yau three-fold with only one two-form (the Kähler form). Then, we will generalize the result to the case where  $M_6$  is a nearly Kähler manifold.

The O6 plane couples to the  $C_7$  flux and sources  $F_2$  magnetically. Therefore, in the presence of O6 plane, the Bianchi identity of  $F_2$  flux becomes

$$d\tilde{F}_2 = -MH_3 - \mu_6\delta_3 \tag{4.38}$$

where  $\delta_3$  is three-form localized on the supersymmetric three-cycle. The smearing of the O6 plane is simply done by replacing  $\mu_6\delta_3$  with  $\mu'_6\text{Re}\Omega$  [36].

For  $M_6$  being  $CY_3$  with only one two-form, we can first start with the most generic flux ansatz, then use the Bianchi identities and equations of motions of the fluxes to eliminate most of the terms. In the end, there are only two possibilities:

$$\begin{aligned}
(1) \quad & H_3 = \beta\theta^{xyr}, \quad \tilde{F}_2 = \alpha\theta^{tr} + \eta\theta^{xy} + aJ, \quad \tilde{F}_4 = f\theta^{txyr} + \frac{s}{2}J^2 + h\theta^{tr} \wedge J + g\theta^{xy} \wedge J \\
(2) \quad & H_3 = k\text{Re}\Omega, \quad \tilde{F}_2 = aJ, \quad \tilde{F}_4 = \frac{s}{2}J^2
\end{aligned} \tag{4.39}$$

The first one reduces to the situation without the O6 plane. Only the second one takes advantage of the presence of the O6 plane, however, it does not break Lorentz symmetry. Therefore, adding an O6 plane in this case does not help finding solution with the

anisotropic scale invariance. This is also clear from (4.35) as it leads to  $z = 1$  by setting  $q = \beta = 0$ . Notice that though the presence of O6 plane changes Einstein equations, it does not change the differences of them such as (4.35).

For  $M_6$  being nearly Kähler, the ineffectiveness of adding an O6 also arises from the fluxes. The flux ansatz remains the same as the one without the O6; the only change is that the second equation coming from the  $F_2$ 's Bianchi identity now becomes

$$ab + km = n_6, \tag{4.40}$$

where  $n_6$  counts the smeared O6 charge after a proper normalization. In the process of solving flux Bianchi identities and equations of motion, the first equation from  $F_4$ 's Bianchi identity and the third equation from  $H_3$ 's equations of motion become

$$\beta n_6 = 0, \quad \text{and} \quad qn_6 = 0, \tag{4.41}$$

respectively. Namely, for a nonzero smeared O6 charge, we have  $q = \beta = 0$ . Again (4.35) requires the relativistic dynamical exponent  $z = 1$ . This means that adding an O6 does not help evade the no-go theorem in the present setup.

## 5 Conclusion and discussion

### 5.1 Conclusions: no-go theorems for Lifshitz-like spacetime in supergravities

In this paper, we examined possibilities of string theory embeddings of the gravity duals (denoted by  $Li_d$ ) for  $(d-1)$ -dimensional quantum Lifshitz-like fixed points, which are invariant under anisotropic scale transformations. We considered the ten- or eleven-dimensional supergravity description of string theory by taking the ordinary low energy limit. A no-go argument in supergravities as general and simple as the one for de-Sitter spaces [28] or the one for flux compactifications [29] does not seem to be available in our case because we consider less symmetric spacetimes with a negative curvature. Therefore, we had to examine each possibilities of compactifications of supergravities.

In all of our supergravity setups (we will summarize them in the next subsection) we examined, we found that it is impossible to construct  $Li_4$  solutions. Notice that in all such setups, there exist  $AdS_4$  solutions. Even though our analysis has been done only for the  $Li_4$  spacetime defined by (2.3), we believe that our result will not significantly lose generality as many aspects of  $AdS_4$  solutions are similar to  $AdS_3$  and  $AdS_5$ . Motivated by these results, we are tempted to conjecture that the gravity duals  $Li_d$  of Lifshitz-like theories cannot be embedded into any supergravity description of string theory or M-theory. The rigorous proof of this no-go theorem will certainly be an important future problem.

Under certain flux ansatze, we proved that no solutions of the form  $Li_4 \times M_7$ , where  $M_7$  is an arbitrary compact manifold, are allowed in the eleven-dimensional supergravity, even if we take the possibility of warp factor into account. We also presented a similar no-go theorem in the massive type IIA supergravity for the spacetime  $Li_4 \times M_6$ . Finally we examined massive IIA compactifications on nearly Kähler manifolds and showed that solutions of the form  $Li_4 \times M_6$  are impossible.

Since the Einstein gravity coupled to a massive vector field can have a Lifshitz-like solution as we noticed in the end of section 2, one may expect that holographic superconductor systems [40, 41] which can be embedded into string theory may have  $Li_d$  solutions.<sup>10</sup> However, we can check after some analysis that  $Li_d$  solutions are impossible as least in the recently found string theory embeddings [42, 43] via ten or eleven dimensional supergravity, as consistent with our claim. In the Chern-Simons-Maxwell type description (2.1) of the gravity dual, our no-go theorem comes from the fact that in the string compactification, the cosmological constant  $\Lambda$  and the Chern-Simons coefficient  $c$  cannot be chosen independently.

As in the case of de-Sitter spaces [44] or flux compactifications [29], one might expect that the addition of orientifolds or non-supersymmetric D-branes may evade our no-go arguments in supergravities. We performed preliminary analysis in tractable examples and observed that such effects do not help us to construct  $Li_d$  spacetimes. Therefore, we do not know at all whether a gravity dual of non-relativistic Lifshitz-like theories exists in string theory or not. We might also improve this situation by considering non-critical string setups such as in [45]. These issues should certainly deserve future studies.

## 5.2 Summary of the no-go setups

Since it might be helpful for future studies of possible Lifshitz-like solutions in supergravities, we would like to summarize our setups where we could manage to prove the no-go theorems for a four dimensional Lifshitz spacetime  $Li_4$  as follows:

- The eleven-dimensional supergravity with the four form flux ansatz given by (3.3) compactified on an arbitrary manifold  $M_7$ . (Section 3)
- The ten-dimensional massive IIA supergravity with the flux ansatz (4.7) compactified on an arbitrary manifold  $M_6$ . (Section 4.2)
- The ten-dimensional massive IIA supergravity with the flux ansatz (4.24) compactified on an nearly Kahler manifold  $M_6$ . (Section 4.3)
- The previous massive IIA nearly Kahler compactifications with orientifold 6-plane. (Section 4.4)

## 5.3 Why string duals of Lifshitz-like points are hard

Before we end this paper, we would like to suggest an intuitive physical reason for our no-go theorems. Consider type IIA or IIB string theory. Let us remember that to realize a  $Li_4$  solution we need to turn on several RR and  $H$  fluxes to break the Lorentz symmetry. The presence of the Chern-Simons coupling, which is important in the argument of [3] as reviewed in section 2, requires turning on an  $H$  flux. Then the requirement  $dH = 0$  argues that  $H \propto \theta^x \theta^y \theta^r$  as is indeed so in our setups.<sup>11</sup> We can easily imagine that such

<sup>10</sup>We would like to thank Gary Horowitz very much for useful discussions on this possibility.

<sup>11</sup>Notice that the solution [14] has vanishing  $H$ -flux. This is the reason why we managed to find the physical solution. However, in this solution the dilaton depends on  $r$  and the metric becomes identical to  $Li_5$  only in the Einstein frame. Thus it is out of the ansatz in our paper since we require the rigorous anisotropic scale invariance.



a gravity solution should be dual to a non-commutative gauge theory as the NS  $B$  field will be induced in the boundary [46]. The effect of non-commutativity is interpreted as the the UV cutoff of the gauge theory. Therefore, the AdS/CFT argues that the IR region is described by AdS, while in the UV region, the spatial directions shrink to zero size. The latter fact requires  $z < 0$  as in the solutions [47] and we will not have a nice scaling limit which extracts the UV part because it contradicts with the relation like (3.18), which requires  $z > 1$ . Note that a simple analysis shows that this condition  $z > 1$  should always be correct even in the presence of the warp factor in the metric  $Li_4 \times M_6$ .

## Acknowledgments

We are very grateful to T. Azeyanagi, T. Banks, G. Giribet, J. Gomis, A. Maloney, J. McGreevy, M. Mulligan, N. Ohta, M. Schulz, D. T. Son, A. Strominger and K. Yoshida for useful discussions, and especially to G. Horowitz, S. Kachru and Y. Nakayama for very important comments. We are supported by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. The work of TN is supported by JSPS Grant-in-Aid for Scientific Research No.19-3589. The work of TT is supported in part by JSPS Grant-in-Aid for Scientific Research No.20740132 and by JSPS Grant-in-Aid for Creative Scientific Research No. 19GS0219. TT would like to thank very much the organizers and participants in the workshop “Quantum Criticality and the AdS/CFT Correspondence”, at KITP in UCSB and those in the conference “Quantum Theory and Symmetries 6” at University of Kentucky, where some parts of our work have been done.

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